

Preface

Satellite Orbits – Models, Methods, and Applications has been written as a comprehensive textbook that guides the reader through the theory and practice of satellite orbit prediction and determination. Starting from the basic principles of orbital mechanics, it covers elaborate force models as well as precise methods of satellite tracking and their mathematical treatment. A multitude of numerical algorithms used in present-day satellite trajectory computation is described in detail, with proper focus on numerical integration and parameter estimation. The wide range of levels provided renders the book suitable for an advanced undergraduate or graduate course on spaceflight mechanics, up to a professional reference in navigation, geodesy and space science. Furthermore, we hope that it is considered useful by the increasing number of satellite engineers and operators trying to obtain a deeper understanding of flight dynamics.

The idea for this book emerged when we realized that documentation on the methods, models and tools of orbit determination was either spread over numerous technical and scientific publications, or hidden in software descriptions that are not, in general, accessible to a wider community. Having worked for many years in the field of spaceflight dynamics and satellite operations, we tried to keep in close touch with questions and problems that arise during daily work, and to stress the practical aspects of orbit determination. Nevertheless, our interest in the underlying physics motivated us to present topics from first principles, and make the book much more than just a cookbook on spacecraft trajectory computation.

With the availability of powerful onground and onboard computers, as well as increasing demands for precision, the need for analytical perturbation theories has almost been replaced by a purely numerical treatment of the equations of motion. We therefore focus on models and methods that can be applied within a numerical reconstruction of the satellite orbit and its forecast. As a consequence, topics like orbit design, long-term orbit evolution and orbital decay are not addressed specifically, although the required fundamentals are provided. Geodesic satellite missions, on the other hand, have reached an unprecedented level of position accuracy with a need for very complex force and measurement models, which could not always be covered in full detail. In any case, references to background information are given, so as to allow the reader easy access to these specific areas.

Each chapter includes exercises at varying levels of complexity, which aim at an additional practice of the presented material, or address supplementary topics of practical interest. Where possible, we have tried to focus on problems that high-

light the underlying physical models or algorithmic methods, rather than relying on purely numerical reference examples. In most cases, the exercises include a comprehensive description of the suggested solution, as well as the numerical results. These are either derived directly from equations given in the text, or based on sample computer programs.

This book comes with a CD-ROM that contains the C++ source code of all sample programs and applications, as well as relevant data files. The software is built around a powerful spaceflight dynamics library, which is likewise provided as source code. For the sake of simplicity we have restricted the library to basic models, but emphasized transparent programming and in-code documentation. This, in turn, allows for an immediate understanding of the code, and paves the way for easy software extensions by the user. Free use of the entire software package including the right for modifications is granted for non-commercial purposes. Readers, students and lecturers are, therefore, encouraged to apply it in further studies, and to develop new applications. We assume that the reader is familiar with computer programming, but even inexperienced readers should be able to use the library functions as black boxes. All source code is written in C++, nowadays a widely used programming language and one which is readily available on a variety of different platforms and operating systems.

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